

What is Claimed:

1 1. A method for performing modular division operations used in a
2 cryptographic process over a finite field F_U defined for a prime number U , in which
3 values are divided by an integer divisor V modulo U , the method comprising the steps
4 of calculating an arithmetic inverse of V modulo U using an extended greatest
5 common divisor (GCD) method which includes a plurality of reduction steps and a
6 plurality of inverse calculations, including the steps of:

7 assigning U and V as initial values to respective temporary variables $U3$
8 and $V3$ which are used to calculate the GCD of U and V ;

9 assigning initial values to respective temporary variables $U2$ and $V2$
10 which are used to calculate an arithmetic inverse of V modulo U ;

11 testing a condition and, if the condition tests true,

12 combining multiple ones of the plurality of reduction steps
13 for calculating the GCD; and

14 combining multiple ones of the plurality of inversion
15 calculations; and

16 if the condition tests false,

17 performing a single one of the reduction steps; and

18 performing a single one of the inverse calculation steps.

1 2. A method according to claim 1, wherein:

2 the extended GCD algorithm is a binary GCD algorithm;

3 the step of testing the condition includes the step of determining if $U3$
4 has a number, N , of zero-valued least significant bits (LSBs), where N is an integer
5 greater than one;

6 the step of combining multiple ones of the plurality of reduction steps
7 includes shifting the value in $U3$ by N bit positions to less significant bit positions;
8 and

9 the step of combining multiple ones of the plurality of inversion
10 calculations includes the steps of:

11 retrieving a value to be combined with V2 from a look-up table
12 responsive to the value of N;

13 combining the retrieved value from V2 to obtain a combined
14 value in V2; and

15 shifting the combined value in V2 by N bit positions to less
16 significant bit positions.

1 3. A method according to claim 2, wherein the look-up table
2 includes a plurality of multiples of U.

1 4. A method according to claim 3, wherein the step of retrieving the
2 value to be combined with V2 from a look-up table includes the steps of:

3 indexing a first further look-up table responsive to two of the LSBs of
4 V2 if N equals 2 to obtain an index value;

5 indexing a second further look-up table responsive to three of the LSBs
6 of V2 if N is greater than 2 to obtain the index value; and

7 indexing the look-up table by the index value.

1 5. A method according to claim 1, wherein:

2 the extended GCD algorithm is a left-shift binary GCD algorithm; and

3 the steps of combining multiple ones of the plurality of reduction steps
4 and combining multiple ones of the plurality of inversion calculations includes the step
5 of performing a reduction step according to a Lehmer GCD method.

1 6. A method according to claim 5, wherein, the step of testing the
2 condition includes the step of determining if a bit position of a most significant bit
3 (MSB) of the value in U3 differs by less than a predetermined number from a bit
4 position of an MSB of the value in V3.

1 7. A computer readable carrier including computer program
2 instructions that cause a computer to perform modular division operations over a finite
3 field F_U that defined for a prime number U and used in a cryptographic process in
4 which values are divided by an integer divisor V modulo U, the method comprising
5 the steps of calculating an arithmetic inverse of V modulo U using an extended

6 greatest common divisor (GCD) method which includes a plurality of reduction steps
7 and a plurality of inverse calculations, including the steps of:

8 assigning U and V as initial values to respective temporary variables U3
9 and V3 which are used to calculate the GCD of U and V;

10 assigning initial values to respective temporary variables U2 and V2
11 which are used to calculate an arithmetic inverse of V modulo U;

12 testing a condition and, if the condition tests true,

13 combining multiple ones of the plurality of reduction steps
14 for calculating the GCD; and

15 combining multiple ones of the plurality of inversion
16 calculations; and

17 if the condition tests false,

18 performing a single one of the reduction steps; and

19 performing a single one of the inverse calculation steps.

1 8. A computer readable carrier according to claim 7, wherein the
2 extended GCD algorithm is a binary GCD algorithm and the computer program
3 instructions which implement the step of testing the condition cause the computer to
4 perform the step of determining if U3 has a number, N, of zero-valued least
5 significant bits (LSBs), where N is an integer greater than one;

6 the computer program instructions which implement the step of
7 combining multiple ones of the plurality of reduction steps cause the computer to
8 perform the step of shifting the value in U3 by N bit positions to less significant bit
9 positions; and

10 the computer program instructions which implement the step of
11 combining multiple ones of the plurality of inversion calculations cause the computer
12 to perform the steps of:

13 retrieving a value to be combined with V2 from a look-up table
14 responsive to the value of N;

15 combining the retrieved value from V2 to obtain a combined
16 value in V2; and

17 shifting the combined value in V2 by N bit positions to less
18 significant bit positions.

1 9. A computer readable carrier according to claim 8, wherein the
2 look-up table includes a plurality of multiples of U.

1 10. A computer readable carrier according to claim 9, wherein the
2 computer program instructions that implement the step of retrieving the value to be
3 combined with V2 from a look-up table cause the computer to perform the steps of:

4 indexing a first further look-up table responsive to two of the LSBs of
5 V2 if N equals 2 to obtain an index value;

6 indexing a second further look-up table responsive to three of the LSBs
7 of V2 if N is greater than 2 to obtain the index value; and

8 indexing the look-up table by the index value.

1 11. A computer readable carrier according to claim 7, wherein the
2 extended GCD algorithm is a left-shift binary GCD algorithm and the computer
3 program instructions that cause the computer to perform the steps of combining
4 multiple ones of the plurality of reduction steps and combining multiple ones of the
5 plurality of inversion calculations includes the step of performing a reduction step
6 according to a Lehmer GCD method.

1 12. A computer readable medium according to claim 11, wherein, the
2 computer program instructions that implement the step of testing the condition cause
3 the computer to perform a step of determining if a bit position of a most significant bit
4 (MSB) of the value in U3 differs by less than a predetermined number from a bit
5 position of a bit position of an MSB of the value in V3.

1 13. Cryptographic apparatus which performs division operations over
2 a finite field F_U defined for a prime number U, in which values are divided by an
3 integer divisor V modulo U, the apparatus calculating an arithmetic inverse of V
4 modulo U using an extended greatest common divisor (GCD) algorithm which
5 includes a plurality of reduction steps and a plurality of inverse calculations, the
6 apparatus comprising:

7 means for assigning U and V as initial values to respective temporary
8 variables U3 and V3 which are used to calculate the GCD of U and V;

9 means for assigning initial values to respective temporary variables U2
10 and V2 which are used to calculate an arithmetic inverse of V modulo U;

11 means for testing a condition; and

12 means for combining multiple ones of the plurality of reduction steps
13 and multiple ones of the inverse calculations if the condition test true;

1 14. Cryptographic apparatus according to claim 13, wherein:

2 the extended GCD algorithm is a binary GCD algorithm;

3 the means for testing the condition includes means for determining if U3
4 has a number, N, of zero-valued least significant bits (LSBs), where N is an integer
5 greater than one;

6 the means for combining multiple ones of the plurality of reduction steps
7 includes means for shifting the value in U3 by N bit positions to less significant bit
8 positions; and

9 the means for combining multiple ones of the plurality of inversion
10 calculations includes:

11 means for retrieving a value to be combined with V2 from a look-
12 up table responsive to the value of N;

13 means for combining the retrieved value from V2 to obtain a
14 combined value in V2; and

15 means for shifting the combined value in V2 by N bit positions to
16 less significant bit positions.

1 15. Apparatus according to claim 14, wherein the look-up table
2 includes a plurality of multiples of U.

1 16. Apparatus according to claim 15, wherein:

2 the means for retrieving the value to be combined with V2 from a look-
3 up table includes:

4 means for indexing a first further look-up table responsive to two of the
5 LSBs of V2 if N equals 2 to obtain an index value;

6 means for indexing a second further look-up table responsive to three of
7 the LSBs of V2 if N is greater than 2 to obtain the index value; and

8 means for indexing the look-up table by the index value.

1 17. Apparatus according to claim 13, wherein:

2 the extended GCD algorithm is a left-shift binary GCD algorithm; and

3 the means for combining multiple ones of the plurality of reduction steps
4 and multiple ones of the plurality of inversion calculations includes means for
5 performing a reduction step according to a Lehmer GCD method.

1 18. Apparatus according to claim 17 wherein, the means for testing
2 the condition includes means for determining if a bit position of a most significant bit
3 (MSB) of the value in U3 differs by less than a predetermined number from a bit
4 position of an MSB of the value in V3.